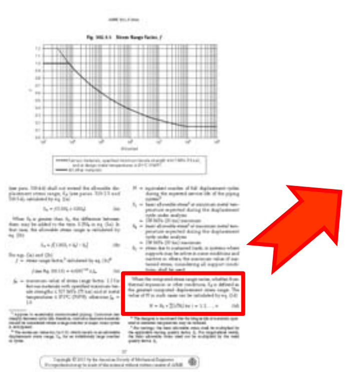
CAU2015

B31.3 302.3.5(d)
"When the computed stress range varies"
– applying existing B31.3 rules in CAESAR II

...and a new piping code proposal:
Allowable Stress for Wave Damage

INTERGRAPH

B31.3 Paragraph 302.3.5(d)
Allowable Displacement Stress Range S_A



When the computed stress range varies, whether from thermal expansion or other conditions, S_E is defined as the greatest computed displacement stress range. The value of N in such cases can be calculated by eq. (1d):

$$N = N_E + \sum(r_i^5 N_i) \text{ for } i = 1, 2, \ldots, n \quad \text{(1d)}$$

where

N_E = number of cycles of maximum computed displacement stress range, S_E

N_i = number of cycles associated with displacement stress range, S_i

r_i = S_i/S_E

S_i = any computed displacement stress range smaller than S_E

INTERGRAPH

Agenda

- Fatigue – Definitions and Use in B31
- Accumulated Damage & Miner's Rule
- Equation (1d)
- Applying (1d)
- Using CAESAR II Fatigue Curve and Accumulated Damage to Satisfy (1d)
- A Worked Example
- A look at a Proposed Code addition providing High Cycle Fatigue Assessment of Piping Systems

INTERGRAPH

FATIGUE

Basics

Fatigue – a Definition*

- Fatigue is the progressive and localized structural damage that occurs when a material is subjected to cyclic loading. The nominal maximum stress values are less than the ultimate tensile stress limit, and may be below the yield stress limit of the material.
- Fatigue occurs when a material is subjected to repeated loading and unloading. If the loads are above a certain threshold, microscopic cracks will begin to form at the surface. Eventually a crack will reach a critical size, and the structure will suddenly fracture. The shape of the structure will significantly affect the fatigue life; square holes or sharp corners will lead to elevated local stresses where fatigue cracks can initiate. Round holes and smooth transitions or fillets are therefore important to increase the fatigue strength of the structure.

* from: http://en.wikipedia.org/wiki/Fatigue_(material)

Fatigue Assessment*

- Fatigue assessment [in the ABS Guide for the Fatigue Assessment of Offshore Structures 2004] relies on the characteristic S-N curve to define fatigue strength under constant amplitude stress and a linear damage accumulation rule (Palmgren-Miner) to define fatigue strength under variable amplitude stress.

* from: COMMENTARY ON THE GUIDE FOR THE FATIGUE ASSESSMENT OF OFFSHORE STRUCTURES (April 2003) JANUARY 2004
(Updated April 2010) – American Bureau of Shipping available at:
http://www.eagle.org/eagleExternalPortalWEB/ShowProperty/BEA%20Repository/Rules&Guides/Current/115_FatigueAssessmentofOffshoreStructures/FAOS_Commentary

Fatigue in B31

- In his 1947 paper*, A.R.C. Markl adopted the following general formula to reflect his fatigue test results:

$$S_N = \frac{245{,}000}{\sqrt[5]{N}}$$

  - Where S_N (in psi) is the endurance strength in terms of the number N of cycles of complete reversal producing failure

- This is an endurance curve

* "Fatigue Tests of Welding Elbows and Comparable Double-Miter Bends" (Transactions of ASME Volume 69)

INTERGRAPH

Fatigue in B31

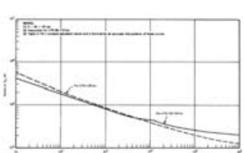
- This S-N curve is expressed in the formula for S_A, the allowable displacement stress range (B31.3 Eqn.(1a)):

$$S_A = f(1.25S_c + 0.25S_h)$$

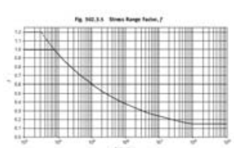
Where, in Eqn.(1c):

$$f = 6.0(N)^{-0.2}$$

Old ASME II Part D S-N curve:



f from B31.3:



INTERGRAPH

Fatigue in B31

- B31.3 paragraph 302.3.5(d) states that the computed displacement stress range, S_E, shall not exceed the allowable displacement stress range, S_A; or:

$$S_E \leq 6.0(N)^{-0.2}(1.25S_c + 0.25S_h)$$

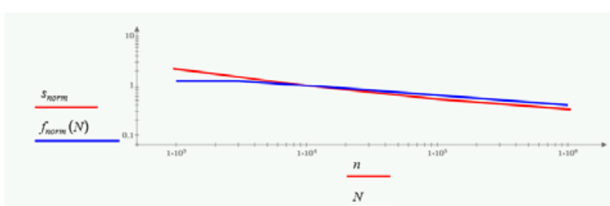
- Compare with Markl:

$$S_E \leq S_N = 245{,}000(N)^{-0.2}$$

INTERGRAPH

Fatigue in B31

- Or, compare a normalized *f* with a normalized polished bar curve (*s*) from the current ASME II-D*:



* Here, normalized means the value equals 1.0 at 10,000 cycles

INTERGRAPH

EVALUATING FATIGUE DAMAGE

Palmgren-Miner Rule

Fatigue Damage*

- Fatigue stress is a random process.
  Stress ranges in the long-term process form a sequence of dependent random variables, S_i; $i = 1, N_T$. For purposes of fatigue analysis and design, it is assumed that S_i are mutually independent. The set of S_i can be decomposed and discretized into J blocks of constant amplitude stress:

Deterministic Stress Spectra

Stress Range S_i	Number of Cycles n_i
S_1	n1
S_2	n2
S_3	n3
…	…
S_{J-1}	n_{J-1}
S_J	n_J

* from: COMMENTARY ON THE GUIDE FOR THE FATIGUE ASSESSMENT OF OFFSHORE STRUCTURES

INTERGRAPH

The Palmgren-Miner Rule defines Fatigue Damage*

- Applying the Palmgren-Miner linear cumulative damage hypothesis to the block loading of the preceding table, cumulative fatigue damage, *D*, is defined as:

$$D = \sum_{i=1}^{J} \frac{n_i}{N_i}$$

where N_i is the number of cycles to failure at stress range S_i, as determined by the appropriate S-N curve.

- Failure is then said to occur if:

$$D > 1.0$$

* ibid

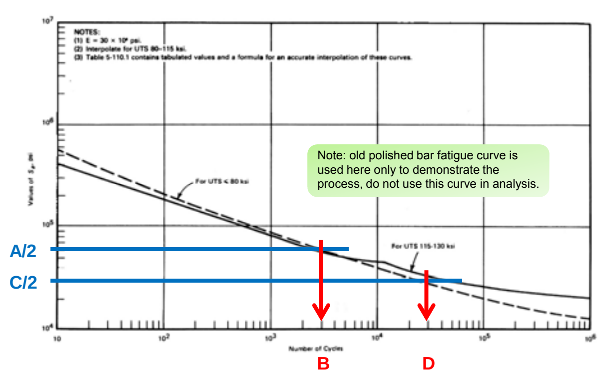
An example of accumulated damage

- For example:

Given		Find
S_{Ei}	N_i	N_{ti}
A	N_1	B
C	N_2	D

- If S_{Ei} is stress range, use $S_{Ei}/2$ as stress amplitude

$$D = \frac{N1}{B} + \frac{N2}{D}$$



Limitations Miner's Rule*

- Though Miner's rule is a useful approximation in many circumstances, it has several major limitations:
  - It fails to recognize the probabilistic nature of fatigue and there is no simple way to relate life predicted by the rule with the characteristics of a probability distribution.
  - There is sometimes an effect in the order in which the reversals occur. In some circumstances, cycles of low stress followed by high stress cause more damage than would be predicted by the rule.

* from: http://en.wikipedia.org/wiki/Miner%27s_rule#Miner.27s_rule

INTERGRAPH

EQUATION (1d)

B31.3 Paragraph 302.3.5(d)
Allowable Displacement Stress Range *SA*

When the computed stress range varies, whether from thermal expansion or other conditions, S_E is defined as the greatest computed displacement stress range. The value of N in such cases can be calculated by eq. (1d):

$$N = N_E + \sum(r_i^5 N_i) \text{ for } i = 1, 2, \ldots, n \qquad \text{(1d)}$$

where

N_E = number of cycles of maximum computed displacement stress range, S_E

N_i = number of cycles associated with displacement stress range, S_i

r_i = S_i/S_E

S_i = any computed displacement stress range smaller than S_E

INTERGRAPH

Deriving Equation (1d) 1/4

- Convert smaller stress ranges into equivalent cycles for the maximum stress range.
- Evaluate the largest calculated expansion stress range against an adjusted allowable limit

- Terms
  - $S_E = maximum\ stress\ range$
  - $S_i = each\ smaller\ stress\ range$
  - $N_E = cycles\ at\ S_E$
  - $N_i = cycles\ at\ S_i$
  - $N_{E\ allowed} = cycles\ allowed\ at\ S_E$
  - $N_{i\ allowed} = cycles\ allowed\ at\ S_i$
  - $N_{i\ equivalent} = cycles\ at\ S_E$
  - $k = Markl's\ material\ constant$

INTERGRAPH

Deriving Equation (1d) 2/4

- Markl says:
  - $S_{allowed} = kN^{-0.2}$

    - or, solving for *N* -

  - $N_{i\ allowed} = \left(k/S_i\right)^5$

- Terms
  - $S_E = maximum\ stress\ range$
  - $S_i = each\ smaller\ stress\ range$
  - $N_E = cycles\ at\ S_E$
  - $N_i = cycles\ at\ S_i$
  - $N_{E\ allowed} = cycles\ allowed\ at\ S_E$
  - $N_{i\ allowed} = cycles\ allowed\ at\ S_i$
  - $N_{i\ equivalent} = cycles\ at\ S_E$
  - $k = Markl's\ material\ constant$

INTERGRAPH

Deriving Equation (1d) 3/4

- The ratio of actual cycles to allowed cycles could be used to determine the number of equivalent cycles for the maximum stress range
  - $\frac{N_i}{N_{i\ allowed}} = \frac{N_{i\ equivalent}}{N_{E\ allowed}}$; or
  - $N_{i\ equivalent} = N_i \cdot \frac{N_{E\ allowed}}{N_{i\ allowed}}$
  - $N_{i\ equivalent} = N_i \cdot \frac{\left(k/S_E\right)^5}{\left(k/S_i\right)^5} = N_i\left(S_i/S_E\right)^5$

- Terms
  - $S_E = maximum\ stress\ range$
  - $S_i = each\ smaller\ stress\ range$
  - $N_E = cycles\ at\ S_E$
  - $N_i = cycles\ at\ S_i$
  - $N_{E\ allowed} = cycles\ allowed\ at\ S_E$
  - $N_{i\ allowed} = cycles\ allowed\ at\ S_i$
  - $N_{i\ equivalent} = cycles\ at\ S_E$
  - $k = Markl's\ material\ constant$

INTERGRAPH

Deriving Equation (1d) 4/4

- with:

$$N_{i\ equivalent} = N_i\left({}^{S_i}/_{S_E}\right)^5$$

- letting:

$$r_i = {}^{S_i}/_{S_E}$$

- gives:

$$N = N_E + \sum(r_i{}^5 N_i)$$

B31.3 :

When the computed stress range varies, whether from thermal expansion or other conditions, S_E is defined as the greatest computed displacement stress range. The value of N in such cases can be calculated by eq. (1d):

$$N = N_E + \sum(r_i^5 N_i) \text{ for } i = 1, 2, \ldots, n \qquad \text{(1d)}$$

where

N_E = number of cycles of maximum computed displacement stress range, S_E

N_i = number of cycles associated with displacement stress range, S_i

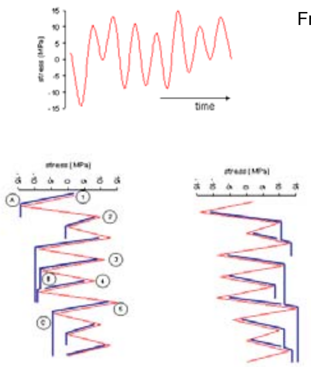
r_i = S_i/S_E

S_i = any computed displacement stress range smaller than S_E

INTERGRAPH

COUNTING CYCLES

A Note on Counting Cycles – Rainflow Counting



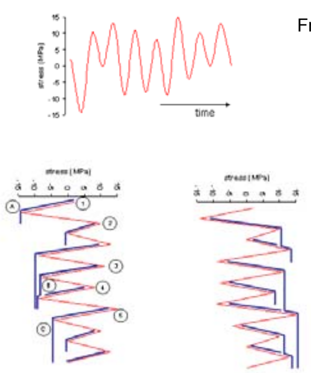
From: http://en.wikipedia.org/wiki/Rainflow-counting_algorithm

The algorithm (Similar to ASME VIII-2 Annex 5-B)

1. Reduce the time history to a sequence of (tensile) peaks and (compressive) troughs.
2. Rotate this sheet clockwise 90° (earliest time to the top).
3. Each tensile peak is imagined as a source of water that "drips" down the pagoda.
4. Count the number of half-cycles by looking for terminations in the flow occurring when either:
    1. It reaches the end of the time history;
    2. It merges with a flow that started at an earlier tensile peak; or
    3. It flows opposite a tensile peak of greater magnitude …

INTERGRAPH

A Note on Counting Cycles – Rainflow Counting



From: http://en.wikipedia.org/wiki/Rainflow-counting_algorithm

The algorithm continued…

5. Repeat step 5 for compressive troughs.
6. Assign a magnitude to each half-cycle equal to the stress difference between its start and termination.
7. Pair up half-cycles of identical magnitude (but opposite sense) to count the number of complete cycles. Typically, there are some residual half-cycles.

See also: TD/12

INTERGRAPH

Counting Cycles – Example

- Given the stress history below, determine the total number of cycles for each stress range
- Note: Start = End = 0

INTERGRAPH

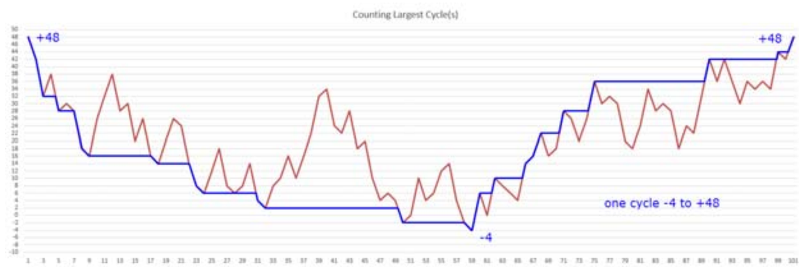
Counting Cycles – Example

- Shift to start with largest stress

INTERGRAPH

Counting Cycles – Example

- Starting with the maximum stress and always moving to the right, track the path to the lowest stress. Then, track the path back to the maximum.
- The path need not be contiguous.



Counting Cycles – Example

- Continue counting

Counting Cycles – Example

- Continue counting

INTERGRAPH

Counting Cycles – Example

- Continue counting

INTERGRAPH

Counting Cycles – Example

- Continue counting

INTERGRAPH

Counting Cycles – Example

- Continue counting

INTERGRAPH

Counting Cycles – Example

- Continue counting

INTERGRAPH

Counting Cycles – Example

- Continue counting

INTERGRAPH

Counting Cycles – Example

- Continue counting

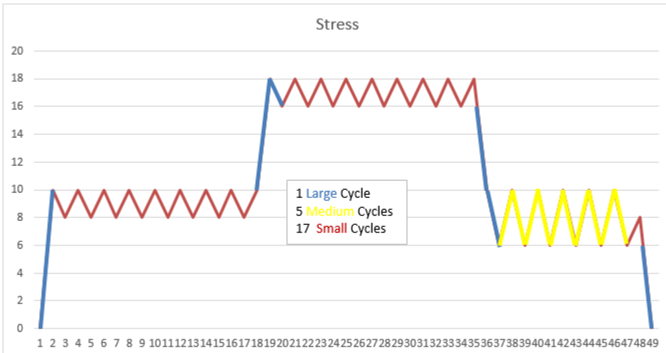
INTERGRAPH

Counting Cycles – Summary

Set	Count	Range	Max	Min
1	1	52	48	-4
2	1	46	44	-2
3	1	42	42	0
4	1	36	38	2
5	1	34	38	4
6	1	20	36	16
7	1	18	36	18
8	1	10	14	4
9	1	8	26	18
10	1	6	26	20
11	1	6	10	4
12	1	2	30	28
13	1	2	24	22

INTERGRAPH

Another quick example



INTERGRAPH

APPLYING (1d)

CAESAR II fatigue evaluation

- CAESAR II offers a more complete fatigue evaluation utilizing cumulative damage as calculated by the Miner's Rule
- A fatigue curve must be provided to relate the stress (amplitude) to the allowed number of cycles, along with
- The expected number of cycles (rather than the "*f*" associated with that number of cycles)
- Where do we collect this S-N fatigue curve?

INTERGRAPH

Using Equation (1d)

- This is difficult to apply!
  - The maximum computed displacement stress range, S_E, at any node can be set by any one of the several calculated stress ranges
  - The individual (lesser) stress ranges, S_i, vary as well

When the computed stress range varies, whether from thermal expansion or other conditions, S_E is defined as the greatest computed displacement stress range. The value of N in such cases can be calculated by eq. (1d):

$$N = N_E + \sum(r_i^5 N_i) \text{ for } i = 1, 2, \ldots, n \qquad (1d)$$

where

N_E = number of cycles of maximum computed displacement stress range, S_E

N_i = number of cycles associated with displacement stress range, S_i

r_i = S_i/S_E

S_i = any computed displacement stress range smaller than S_E

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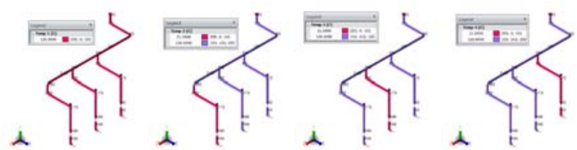
INTERGRAPH

Using Equation (1d)

- Example: Pump manifold, analyze all hot and one spared pump.



INTERGRAPH

Using Equation (1d)

- Example: Pump manifold, analyze all hot and one spared pump.

  This gives 4 operating states and 10 expansion ranges:

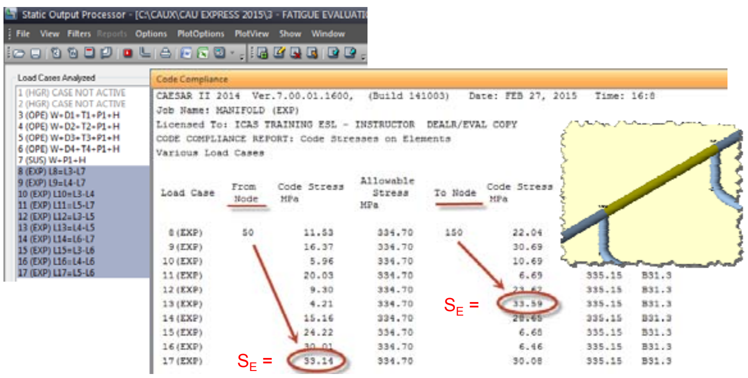
Load Cases Analyzed
1 (HGR) CASE NOT ACTIVE
2 (HGR) CASE NOT ACTIVE
3 (OPE) W+D1+T1+P1+H
4 (OPE) W+D2+T2+P1+H
5 (OPE) W+D3+T3+P1+H
6 (OPE) W+D4+T4+P1+H
7 (SUS) W+P1+H
8 (EXP) L8=L3-L7
9 (EXP) L9=L4-L7
10 (EXP) L10=L3-L4
11 (EXP) L11=L5-L7
12 (EXP) L12=L3-L5
13 (EXP) L13=L4-L5
14 (EXP) L14=L6-L7
15 (EXP) L15=L3-L6
16 (EXP) L16=L4-L6
17 (EXP) L17=L5-L6

8	All Hot – All Ambient	10	All Hot – Left Ambient	13	Left Ambient – Center Ambient	17	Center Ambient – Right Ambient
9	Left Ambient – All Ambient	12	All Hot – Center Ambient	16	Left Ambient – Right Ambient		
11	Center Ambient – All Ambient	15	All Hot – Right Ambient				
14	Right Ambient – All Ambient						

Note that CAESAR II now automatically creates ("recommends") all 10 ranges

INTERGRAPH

Different Maxima


Static Output Processor - [C:\CAUX\CAU EXPRESS 2015\3 - FATIGUE EVALUATI

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Load Cases Analyzed

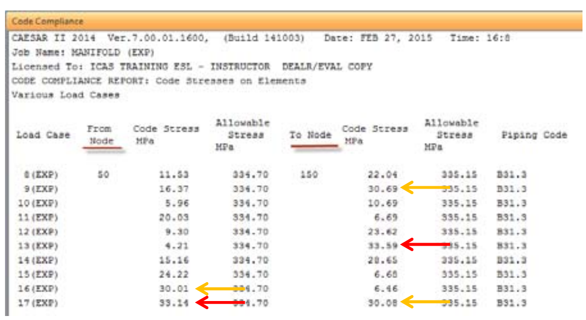
1 (HGR) CASE NOT ACTIVE
2 (HGR) CASE NOT ACTIVE
3 (OPE) W+D1+T1+P1+H
4 (OPE) W+D2+T2+P1+H
5 (OPE) W+D3+T3+P1+H
6 (OPE) W+D4+T4+P1+H
7 (SUS) W+P1+H
8 (EXP) L8=L3-L7
9 (EXP) L9=L4-L7
10 (EXP) L10=L3-L4
11 (EXP) L11=L5-L7
12 (EXP) L12=L3-L5
13 (EXP) L13=L4-L5
14 (EXP) L14=L6-L7
15 (EXP) L15=L3-L6
16 (EXP) L16=L4-L6
17 (EXP) L17=L5-L6

Code Compliance

CAESAR II 2014 Ver.7.00.01.1600, (Build 141003) Date: FEB 27, 2015 Time: 16:8
Job Name: MANIFOLD (EXP)
Licensed To: ICAS TRAINING ESL - INSTRUCTOR DEALR/EVAL COPY
CODE COMPLIANCE REPORT: Code Stresses on Elements
Various Load Cases

Load Case	From Node	Code Stress MPa	Allowable Stress MPa	To Node	Code Stress MPa		
8 (EXP)	50	11.53	334.70	150	22.04		
9 (EXP)		16.37	334.70		30.69		
10 (EXP)		5.96	334.70		10.69		
11 (EXP)		20.03	334.70		6.69	335.15	B31.3
12 (EXP)		9.30	334.70		23.62	335.15	B31.3
13 (EXP)		4.21	334.70		33.59	335.15	B31.3
14 (EXP)		15.16	334.70		28.65	335.15	B31.3
15 (EXP)		24.22	334.70		6.68	335.15	B31.3
16 (EXP)		30.01	334.70		6.46	335.15	B31.3
17 (EXP)		33.14	334.70		30.08	335.15	B31.3

Several Ranges are Significant


Code Compliance

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CODE COMPLIANCE REPORT: Code Stresses on Elements
Various Load Cases

Load Case	From Node	Code Stress MPa	Allowable Stress MPa	To Node	Code Stress MPa	Allowable Stress MPa	Piping Code
8 (EXP)	50	11.53	334.70	150	22.04	335.15	B31.3
9 (EXP)		16.37	334.70		30.69	335.15	B31.3
10 (EXP)		5.96	334.70		10.69	335.15	B31.3
11 (EXP)		20.03	334.70		6.69	335.15	B31.3
12 (EXP)		9.30	334.70		23.62	335.15	B31.3
13 (EXP)		4.21	334.70		33.59	335.15	B31.3
14 (EXP)		15.16	334.70		28.65	335.15	B31.3
15 (EXP)		24.22	334.70		6.68	335.15	B31.3
16 (EXP)		30.01	334.70		6.46	335.15	B31.3
17 (EXP)		33.14	334.70		30.08	335.15	B31.3

Several Ranges are Significant

- Applying (1d) at Node 150:

	A	B	C	D	E	F	G
1			Review of Node 150				
2	L/C	N	Stress	r=Si/SE	r^5	(r^5)*N	
3	8	300	22.04	0.656	0.122	36	
4	9	3000	30.69	0.914	0.637	1910	
5	10	100	10.69	0.318	0.003	0	
6	11	3000	6.69	0.199	0.000	1	
7	12	100	23.62	0.703	0.172	17	
8	13	10000	33.59	1.000	1.000	10000	
9	14	3000	28.65	0.853	0.451	1354	
10	15	100	6.68	0.199	0.000	0	
11	16	10000	6.46	0.192	0.000	3	
12	17	10000	30.08	0.896	0.576	5759	
13		SE:	33.59		N :	19081	
14							
15		N	f				
16		10000	0.95	: f for LC13 alone			
17		19081	0.84	: updated f for other, equivalent cycles			

INTERGRAPH

Checking Node 150

- Load Case 13 sets the expansion stress range $S_E = 33.59\ MPa$
- Apply (1d): $N = N_E + \sum(r_i{}^5 N_i)$
- *f* changes from:
  - 0.95 (10,000 cycles)

  to:
  - 0.84 (19,081 cycles)
- Allowable stress drops by 12%
- No other expansion stress ranges require evaluation for this node

When the computed stress range varies, whether from thermal expansion or other conditions, S_E is defined as the greatest computed displacement stress range. The value of N in such cases can be calculated by eq. (1d):

$$N = N_E + \sum(r_i^5 N_i) \text{ for } i = 1, 2, \ldots, n \qquad \text{(1d)}$$

where

N_E = number of cycles of maximum computed displacement stress range, S_E

N_i = number of cycles associated with displacement stress range, S_i

r_i = S_i/S_E

S_i = any computed displacement stress range smaller than S_E

INTERGRAPH

USING CAESAR II TO SATISFY (1d)

Fatigue in B31

- We will use B31.3 allowable displacement stress range equation (1a) as our fatigue curve in CAESAR II:

$$S_A = f(1.25S_c + 0.25S_h)$$

- but

$$f = 6.0(N)^{-0.2}$$

- so

$$S_A = 6.0(N)^{-0.2}(1.25S_c + 0.25S_h)$$

- Equation (1b) is not as conservative but it includes the (perhaps varying) longitudinal stress due to sustained loads:

?

$$S_A = 6.0(N)^{-0.2}[1.25(S_c + S_h) - S_L]$$

INTERGRAPH

Fatigue Curve in CAESAR II

Excel Calculation

Create a C2 Fatigue curve to reflect Markl

use Eqn (1a)

Sh = 20 ksi

Sc = 20 ksi

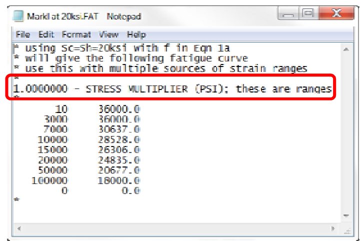
f = 6N^-0.2

fmax = 1.2

SA = f(1.25Sc+0.25Sh)

N (x1000)	f	SA (psi)
0.01	1.20	36000
3	1.20	36000
7	1.02	30637
10	0.95	28528
15	0.88	26306
20	0.83	24835
50	0.69	20677
100	0.60	18000

CAESAR II Data File


Markl at 20ksi.FAT Notepad

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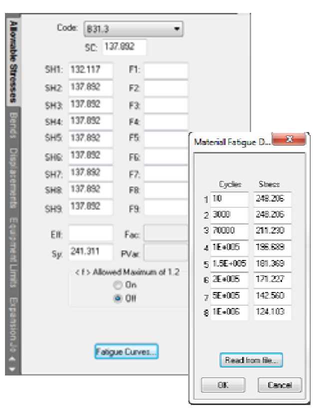
```
* using Sc=Sh=20ksi with f in Eqn 1a
* will give the following fatigue curve
* use this with multiple sources of strain ranges
*
1.0000000 - STRESS MULTIPLIER (PSI); these are ranges
*
       10      36000.0
     3000      36000.0
     7000      30637.0
    10000      28528.0
    15000      26306.0
    20000      24835.0
    50000      20677.0
   100000      18000.0
        0          0.0
*
```

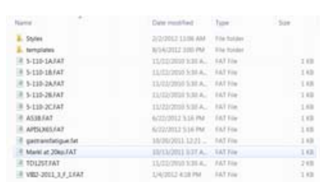
Stored in the SYSTEM folder, CAESAR II will use these data to establish the fatigue curve in the analysis. Note the "Stress Multiplier" is set to 1.0 rather than the 0.5 found in other FAT files. We are indicating a range evaluation here, rather than the typical amplitude values in S-N curves.

INTERGRAPH

Entering the Fatigue Curve (setting *S*)

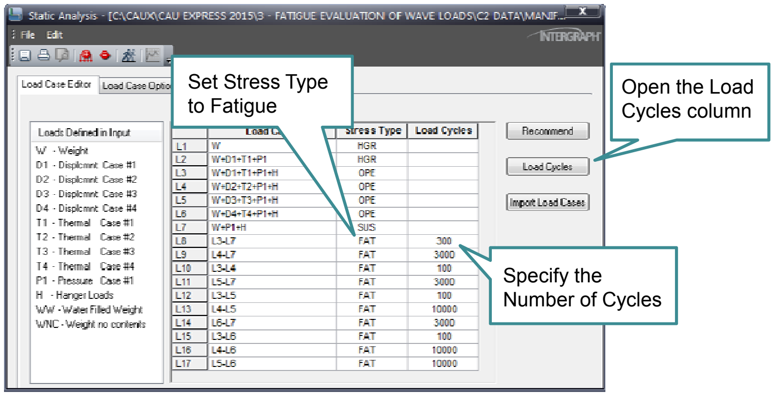


1. In the Allowable Stress window, click on "Fatigue Curves" to open the dialog
2. "Read from file"
3. Select the fatigue file



INTERGRAPH

Load Cases for Fatigue Evaluation (setting *N*)

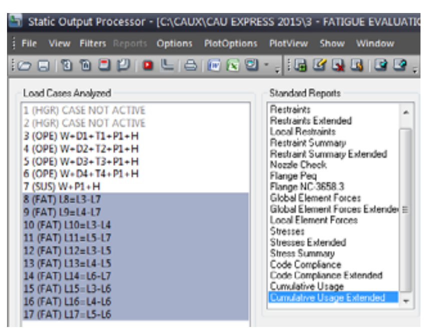


INTERGRAPH

Accumulated damage is calculated in the output processor

- Select all fatigue cases with the Cumulative Usage Report
- CAESAR II will calculate and sum all the selected $N_{demand}/N_{allowed}$ ratios
- OK, if the sum $D < 1$

$$D = \sum_{i=1}^{J} \frac{n_i}{N_i} \overset{?}{<} 1$$



INTERGRAPH

A WORKED EXAMPLE

Comparing (1d) with CAESAR II fatigue evaluation

Worked Example

- Compare the cumulative damage approach (Markl fatigue curve) with the hand application of Equation (1d)
- CAESAR II model: SEVERAL STRAINS
  - A 3 meter cantilever of 4 inch STD A106B pipe
  - Anchored at one end (10)
  - Three imposed lateral displacements at the far end:
    - D1: 39mm, N:14,500 cycles and N:15,000 cycles
    - D2: 38mm, N:14,500 cycles
    - D3: 36.5mm, N:14,500 cycles

INTERGRAPH

Calculate Stresses

- What is the stress range (at node 10, the anchor) for each of the three imposed displacements:

	Displacement at 20 (mm)	Stress Range (MPa)
D1	39.0	150.73
D2	38.0	146.86
D3	36.5	141.07

INTERGRAPH

Calculate *N* using (1d) (with 14,500 for each set)

- S_E is the largest stress range.
  Here, S_E = 150.73 MPa (the first load set)

i	Stress Range (MPa)	N	r_i (=S_i/S_E)	r_i^5	$r_i^5.N_i$
	150.73	14,500	1	1	14,500
1	146.86	14,500	0.974	0.878	12,732
2	141.07	14,500	0.936	0.718	10,412

- $N = N_E + \sum(r_i^5 N_i)$
- *N* = 14500+12732+10412 = 37644

INTERGRAPH

Calculate S_A (using 1a) and Evaluate

- $S_A = f(1.25S_c + 0.25S_h)$
- $f = 6.0(N)^{-0.2} = 6.0(37644)^{-0.2} = 0.73$
- $S_c = S_h = 137.892\ MPa$
- $S_A = 150.88\ MPa$
- $S_E = 150.73\ MPa$
- $S_E \leq S_A$ ✓

INTERGRAPH

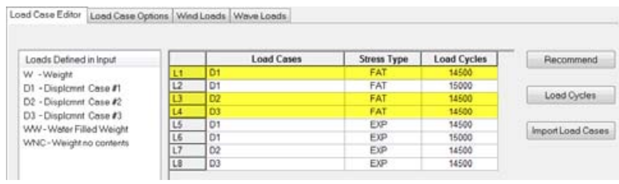
Simple but Tedious

- No single expansion stress range will always produce the maximum stress range S_E
- Stress ratios will vary between load cases and vary from node to node
- An accounting headache!

INTERGRAPH

Using the CAESAR II Fatigue Curve & Accumulated Damage

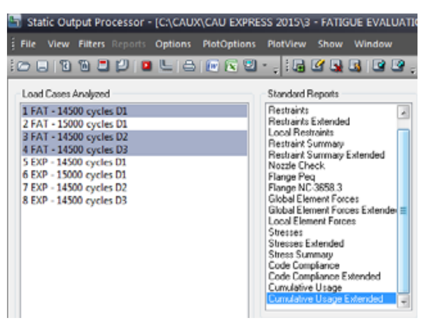
- The example fatigue curve reviewed earlier, MARKL AT 20KSI.FAT, matches the allowable stress range equation (1a)
- The appropriate number of cycles was defined in the Load Case Editor. Note that the larger imposed displacement (D1) is entered twice, we will use the first entry, *N*=14500, now:



INTERGRAPH

Using the CAESAR II Fatigue Curve & Accumulated Damage

- Select the proper set of loads to evaluate:



INTERGRAPH

Using the CAESAR II Fatigue Curve & Accumulated Damage

- View the results:

Cumulative Usage Extended

CAESAR II 2014 Ver.7.00.01.1600, (Build 141003) Date: FEB 24, 2015 Time: 17:0
Job Name: SEVERAL STRAINS
Licensed To: ICAS TRAINING ESL - INSTRUCTOR DEALR/EVAL COPY

CAESAR II CUMULATIVE USAGE

Load Case	Cycles	From Node	(MPa	Stress)	Allowable Cycles	Usage Ratio	To Node	(MPa	Stress)	Allowable Cycles	Usage Ratio
CASE 1 FAT - 14500 cycles D1	14500	10		150.73	37841	0.38	20		0.00	INFINITY	0.00
CASE 3 FAT - 14500 cycles D2	14500	10		146.86	43090	0.34	20		0.00	INFINITY	0.00
CASE 4 FAT - 14500 cycles D3	14500	10		141.07	52703	0.28	20		0.00	INFINITY	0.00
TOTAL:		10				0.99	20				0.00

Load Case Information | Results for Node 10 | Results for Node 20

INTERGRAPH

Using the CAESAR II Fatigue Curve & Accumulated Damage

- Node 10 details:

Cumulative Usage Extended

CAESAR II 2014 Ver.7.00.01.1600, (Build 141003) Date: FEB 24, 2015 Time: 17:0
Job Name: SEVERAL STRAINS
Licensed To: ICAS TRAINING ESL - INSTRUCTOR DEALR/EVAL COPY

CAESAR II CUMULATIVE USAGE

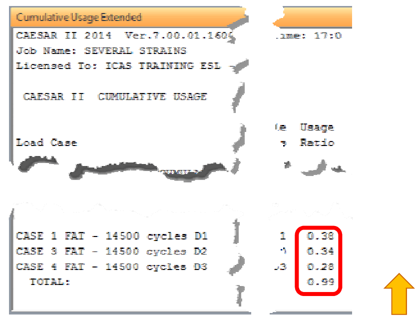
Load Case	Cycles	From Node	(MPa	Stress)	Allowable Cycles	Usage Ratio
CASE 1 FAT - 14500 cycles D1	14500	10		150.73	37841	0.38
CASE 3 FAT - 14500 cycles D2	14500	10		146.86	43090	0.34
CASE 4 FAT - 14500 cycles D3	14500	10		141.07	52703	0.28
TOTAL:		10				0.99

- Allowable Cycles comes from fatigue curve (given *S*, find *N*)
- Usage Ratio is (Cycles Required)/(Cycles Allowed)
- If the sum of ratios is < 1, fatigue is within limits

INTERGRAPH

Compare Results

- The (1d) "hand" calculation resets the number of cycles used for the highest stress.
  - S_E at Node 10=max(150.73, 146.86, 141.07)=150.73 MPa
  - $N_{equivalent}$ = 37644, therefor f = 0.729
  - S_A = 150.88 MPa
  - $S_E < S_A$ ✓
- The CAESAR II Accumulated Damage report collects fatigue damage for each stress range.
  - 0.383+0.336+0.275 = 0.994 < 1 ✓



Reworked Example

- Now, for the existing system and loads, adjust the number of cycles:

	Displacement at 20 (mm)	Stress Range (MPa)	N previous	N now
D1	39.0	150.73	14,500	15,000
D2	38.0	146.86	14,500	14,500
D3	36.5	141.07	14,500	14,500

Recalculate N

- S_E is the largest stress range.
  Here, S_E = 150.73 MPa (the first load set)

i	Stress Range (MPa)	N	r_i (=S_i/S_E)	r_i^5	$r_i^5 \cdot N_i$
	150.73	15,000	1	1	15,000
1	146.86	14,500	0.974	0.878	12,732
2	141.07	14,500	0.936	0.718	10,412

- $N = N_E + \sum(r_i{}^5 N_i)$
- N = 15000+12732+10412 = 38144

INTERGRAPH

Recalculate SA (1a) and Evaluate

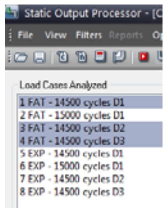
- $S_A = f(1.25S_c + 0.25S_h)$
- $f = 6.0(N)^{-0.2} = 6.0(38144)^{-0.2} = 0.728$
- $S_c = S_h = 137.892\ MPa$
- $S_A = 150.486\ MPa$
- $S_E = 150.73\ MPa$
- $S_E \leq S_A$ ×

INTERGRAPH

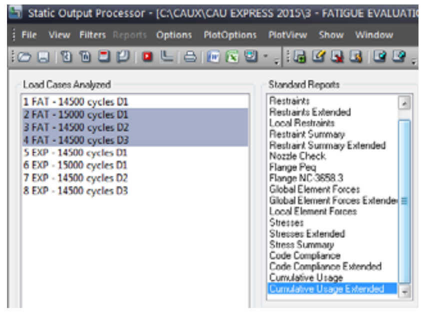
Using the CAESAR II Fatigue Curve & Accumulated Damage

- Select the proper set of loads to evaluate:

From:



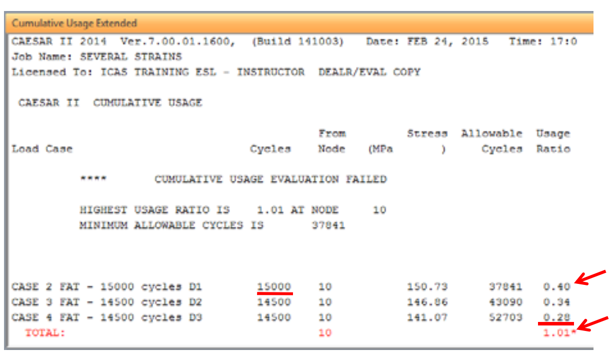
To:



INTERGRAPH

Using the CAESAR II Fatigue Curve & Accumulated Damage

- Node 10 details:



```
Cumulative Usage Extended
CAESAR II 2014  Ver.7.00.01.1600,  (Build 141003)   Date: FEB 24, 2015   Time: 17:0
Job Name: SEVERAL STRAINS
Licensed To: ICAS TRAINING ESL - INSTRUCTOR  DEALR/EVAL COPY

 CAESAR II  CUMULATIVE USAGE

                                         From         Stress  Allowable  Usage
Load Case                        Cycles  Node   (MPa       )     Cycles  Ratio

          ****       CUMULATIVE USAGE EVALUATION FAILED

          HIGHEST USAGE RATIO IS    1.01 AT NODE     10
          MINIMUM ALLOWABLE CYCLES IS      37841


CASE 2 FAT - 15000 cycles D1      15000   10          150.73     37841   0.40
CASE 3 FAT - 14500 cycles D2      14500   10          146.86     43090   0.34
CASE 4 FAT - 14500 cycles D3      14500   10          141.07     52703   0.28
  TOTAL:                                  10                             1.01*
```

  - With a higher cycle count, D1 Usage Ratio changes from 0.38 to 0.40
  - Accumulated Damage now greater than 1.0

INTERGRAPH

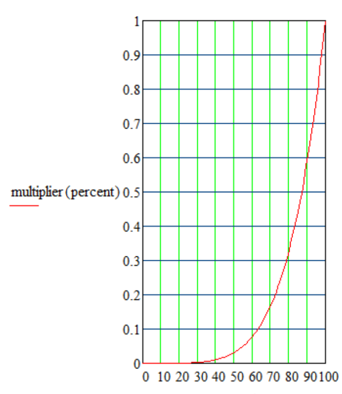
Conclusion

- "When the computed stress range varies" the CAESAR II fatigue evaluation (by accumulated damage) is equivalent to the application of B31.3 equation (1d).
- Accumulated damage is automatic in CAESAR II provided the proper fatigue curve is used and all expected cycle sets are counted.
- Accumulated damage evaluation in CAESAR II is simpler to apply than equation (1d).

INTERGRAPH

Is All This Important?

- Remember, the cycle count is adjusted by the stress ratio to the 5th power:
  - $N = N_E + \sum (S_i/S_E)^5 N_i$
- The multiplier drops rapidly with the ratio S_i/S_E:
  - ratio=.8, increase N by 30% N_i
  - ratio=.6, use <10% of N_i



INTERGRAPH

Questions / Discussion?

INTERGRAPH

A LOOK AT A PROPOSED CODE CHANGE TO ACCOMMODATE WAVE LOADS

Appendix W

Source of this Material

- Commentary on the Guide for the Fatigue Assessment of Offshore Structures (2003) Updated April 2010 – American Bureau of Shipping
- Related / companion documents
  - Guide for the Fatigue Assessment of Offshore Structures (2003) Updated April 2010 – American Bureau of Shipping
  - DNV-RP-C203 Fatigue Design of Offshore Steel Structures (with Commentary)

INTERGRAPH

DEFINING ACCUMULATED FATIGUE DAMAGE

Remaining life for wave loads

- Consider the accumulated fatigue damage in groups:

$$D = \sum_{j=1}^{J} \frac{N_j}{N_{tj}} + \sum_{k=1}^{K} \frac{N_k}{N_{tk}} = d_t + d_w$$

- where j represents the stress range-cycle pairs related to displacement loading and k represents stress range-cycle pairs related to wave loading
- Calculate d_t as above to set remaining life (available damage) for wave loading:

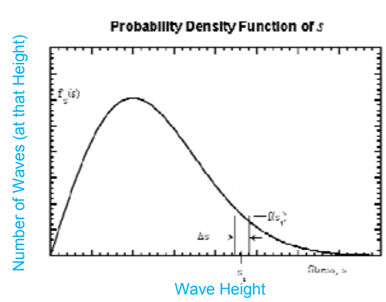
$$d_t = \sum \frac{N_i}{N_{ti}}$$

- Since total damage must remain below 1.0, and with no fatigue design factor, the allowable fatigue damage for variable wave loading would then be:

$$d_w = 1 - d_t \qquad \text{(W-5)}$$

Wave loads are not discrete

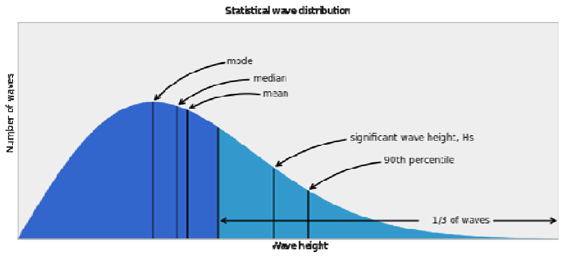
- Fatigue damage due to wave loading is proportional to wave height (trough to peak). Wave height is random, not discrete; one would not count the number of cycles (N_i) for such random stress levels. Wave data often appears, instead, as a Probability Density Function (PDF)

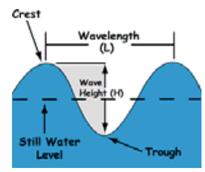


(Modified Figure 2 from ABS Commentary)

Wave Terms

- From the Office of Naval Research
- http://www.onr.navy.mil/focus/ocean/motion/waves1.htm



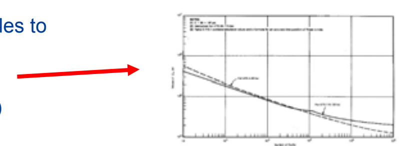


- "Wavestats" by NOAA - NOAA UCAR COMET ProgramRegenerated using python matplotlib and illustrator. Licensed under Public Domain via Wikimedia Commons - http://commons.wikimedia.org/wiki/File:Wavestats.svg#mediaviewer/File:Wavestats.svg

Wave loads are random and continuous

- The fatigue curve will give the number of cycles to failure N_{ti} at stress level s_i can be written as:

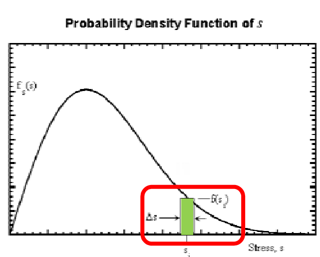
$$N_{ti} = N(s_i) \qquad (5.6)$$



- But now the number of cycles, N_i, grouped around stress level s_i, using the PDF, is the based on the area under the PDF curve:

$$N_i = N_R[f_s(s_i)\Delta s] \qquad (5.7)$$

- Where: N_R is any reference life and $[f_s(s_i)\Delta s]$ is the fraction of the total number of cycles associated with s_i



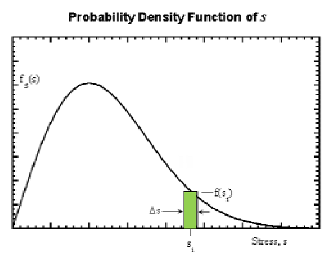
Integrating…

- Substituting this N_i into the summation above gives:

$$D_R = \sum \frac{N_R \cdot f_s(s_i)}{N(s_i)} \Delta s \qquad (5.8) \qquad \left(D = \sum_{i=1}^{J} \frac{\text{Cycles at a stress level}}{\text{Allowed cycles for that stress level}}\right)$$

- D_R is the total wave damage over a reference life N_R.
- The limit, as the group of stresses (Δs) around s_i goes to zero:

$$D_R = N_R \int_0^{\infty} \frac{f_s(s)}{N(s)} ds \qquad (5.9)$$



INTERGRAPH

Accumulated, random damage

$$D_R = N_R \int_0^{\infty} \frac{f_s(s)}{N(s)} ds$$

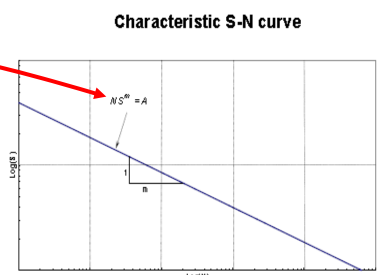
- The number of cycles to failure *N(s)* is set by the fatigue curve:

$$N(s) = A \cdot s^{-m} \qquad (5.10)$$

- Replacing *N(s)* in (5.9) above, we now have the accumulated damage over a life N_R as:

$$D_R = \frac{N_R}{A} \int_0^{\infty} s^m f_s(s)\, ds \qquad (5.11)$$

- Again, D_R is the (reference) damage associated with N_R (reference) cycle life.

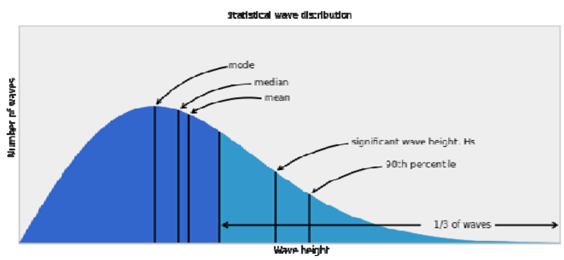


INTERGRAPH

How can this random distribution of stress levels be quantified?

$$D_R = \frac{N_R}{A}\int_0^\infty s^m f_s(s)\, ds$$

- Assume that the stress level produced by wave load is directly proportional to wave height. Historic wave data for certain bodies of water (e.g., North Sea and Gulf of Mexico) show a Weibull distribution of the number of waves at a certain height.



INTERGRAPH

WEIBULL DISTRIBUTION

Two-parameter Weibull Distribution

- Let *S* be a random variable denoting a single stress range associated with wave height in a long-term wave history.
- Assume that *S* has a two-parameter Weibull distribution. The probability that the random variable stress range, *S*, is less than or equal to a certain stress level *s* is:

$$F_S(s) = P(S \leq s) = 1 - e^{-\left(\frac{s}{q}\right)^h} \quad (5.1)$$

  - This is a cumulative distribution function
  - *h* and *q* are the Weibull shape and Weibull scale parameters, respectively.

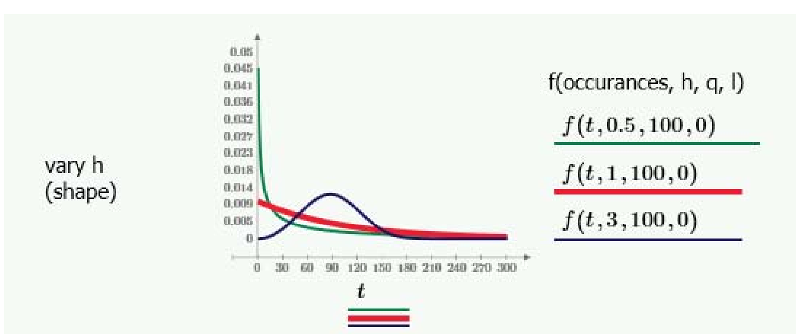
INTERGRAPH

Weibull terms - Shape

- Here are some examples of the Weibull distribution:

exercising the Weibull Distribution (weibull.com):
h is the Weibull Shape Parameter
q is the Weibull Scale Parameter
l is the Weibull Location Parameter (not applicable to wave loading)

$$f(t,h,q,l) := \frac{h}{q}\cdot\left(\frac{t-l}{q}\right)^{h-1}\cdot e^{-\left(\frac{t}{q}\right)^h} \qquad t := 0,1..1000$$



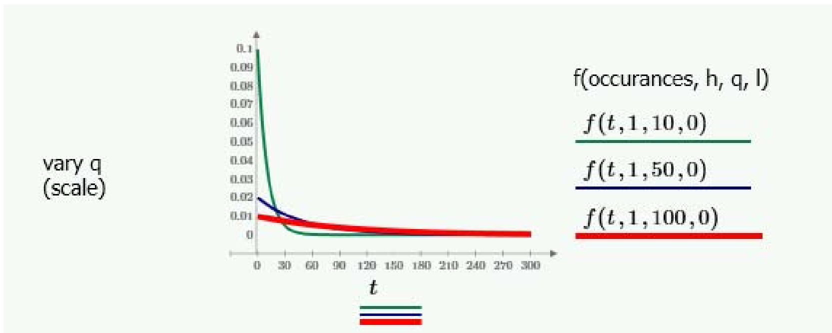
INTERGRAPH

Weibull terms - Scale

- Here are some examples of the Weibull distribution:

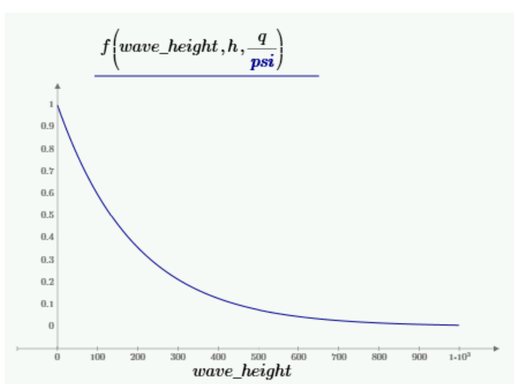
exercising the Weibull Distribution (weibull.com):
h is the Weibull Shape Parameter
q is the Weibull Scale Parameter
l is the Weibull Location Parameter (not applicable to wave loading)

$$f(t,h,q,l) := \frac{h}{q} \cdot \left(\frac{t-l}{q}\right)^{h-1} \cdot e^{-\left(\frac{t}{q}\right)^{h}} \qquad t := 0,1..1000$$



Appendix W offers a default shape parameter

- Here is the Weibull probability distribution where *h*=1.0 (the value mentioned in Appendix W to represent the shape distribution for a typical sea state):



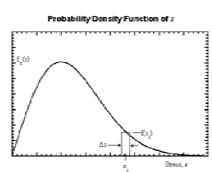
- This plot shows that there are many, many more occurrences of low stress ranges (small waves) than there are high stress ranges (big waves).
- In Appendix W, stress range is assumed directly proportional to wave height but keep in mind that h indicates Weibull shape parameter and not wave height.

WAVE DAMAGE

Wave damage

- Using the cumulative distribution function $F_s(s)$, the number of cycles at stress level *s* is:

$$f_s(s) = \frac{dF_s}{ds} \qquad \text{(para. 5.3)}$$



- With $F_s(s) = 1 - e^{-\left(\frac{s}{q}\right)^h}$:

$$f_s(s) = \left(\frac{h}{q}\right)\left(\frac{s}{q}\right)^{h-1} e^{-\left(\frac{s}{q}\right)^h} \qquad (5.17)$$

$$D_R = \frac{N_R}{A} \int_0^\infty s^m f_s(s)\, ds \qquad (5.11)$$

- Integrating (5.11), the damage at <u>design life</u> (replacing reference life N_R with design life N_d) is:

$$D = \frac{N_d}{A} q^m \Gamma\left(\frac{m}{h} + 1\right) \qquad (5.19)$$

Please do not ask for a derivation of (5.19).

: with the gamma function Γ(.) defined as:

$$\Gamma(x) = \int_0^\infty t^{x-1} e^{-t} dt \qquad (5.3)$$

INTERGRAPH

Evaluating the Gamma Function

$$D = \frac{N_d}{A} q^m \Gamma(\frac{m}{h} + 1)$$

1+m/h	Γ(1+m/h)	1+m/h	Γ(1+m/h)	1+m/h	Γ(1+m/h)	1+m/h	Γ(1+m/h)
3.00	2.00	4.00	6.00	5.00	24.00	6.00	120.00
3.05	2.10	4.05	6.39	5.05	25.88	6.05	130.72
3.10	2.20	4.10	6.81	5.10	27.93	6.10	142.45
3.15	2.31	4.15	7.27	5.15	30.16	6.15	155.31
3.20	2.42	4.20	7.76	5.20	32.58	6.20	169.41
3.25	2.55	4.25	8.29	5.25	35.21	6.25	184.86
3.30	2.68	4.30	8.86	5.30	38.08	6.30	201.81
3.35	2.83	4.35	9.47	5.35	41.20	6.35	220.41
3.40	2.98	4.40	10.14	5.40	44.60	6.40	240.83
3.45	3.15	4.45	10.85	5.45	48.30	6.45	263.26
3.50	3.32	4.50	11.63	5.50	52.34	6.50	287.89
3.55	3.51	4.55	12.47	5.55	56.75	6.55	314.95
3.60	3.72	4.60	13.38	5.60	61.55	6.60	344.70
3.65	3.94	4.65	14.37	5.65	66.80	6.65	377.42
3.70	4.17	4.70	15.43	5.70	72.53	6.70	413.41
3.75	4.42	4.75	16.59	5.75	78.78	6.75	453.01
3.80	4.69	4.80	17.84	5.80	85.62	6.80	496.61
3.85	4.99	4.85	19.20	5.85	93.10	6.85	544.61
3.90	5.30	4.90	20.67	5.90	101.27	6.90	597.49
3.95	5.64	4.95	22.27	5.95	110.21	6.95	655.77

INTERGRAPH

Setting the Weibull Scale parameter

- But the total damage calculation also requires the Weibull scale parameter *q*:

$$D = \frac{N_d}{A} q^m \Gamma(\frac{m}{h} + 1) \qquad (5.19)$$

- This parameter also appears in the cumulative distribution function:

$$F_s(s) = P(S \leq s) = 1 - e^{-\left(\frac{s}{q}\right)^h} \qquad (5.1)$$

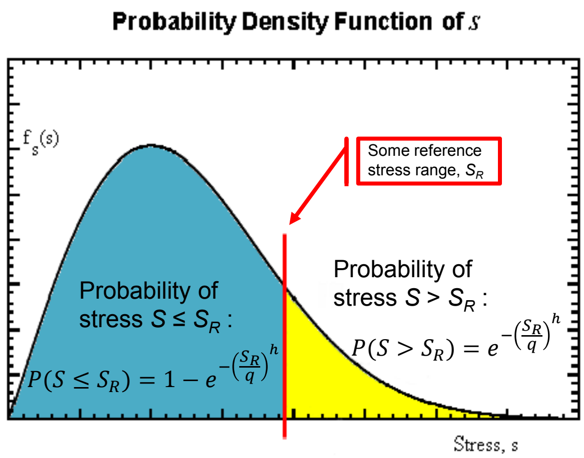
  - This is the probability that a single stress level (*S*) is equal to or <u>below</u> a stress level *s*.

- This function could be rewritten to determine the probability that a stress level (*S*) is <u>above</u> some value *s*, as in:

$$F_s(s) = P(S > s) = e^{-\left(\frac{s}{q}\right)^h}$$

INTERGRAPH

Illustrating this probability



INTERGRAPH

Setting the Weibull Scale parameter

- The probability of a stress level S exceeding a reference stress level, S_R, is:

$$P(S > S_R) = e^{-\left(\frac{S_R}{q}\right)^h}$$

- The "100 year storm" can be used to set this probability where the reference stress level is based on the "100 year storm" wave height.
- By definition, this wave height would be reached once in 100 years, or, in N_w cycles.
- So the reference stress level – the stress associated with the 100 year storm height – will occur once every N_W cycles:

$$P(S > S_R) = e^{-\left(\frac{S_R}{q}\right)^h} = \frac{1}{N_w}$$

- Solving for the Weibull scale parameter:

$$q = \frac{S_R}{(ln(N_w))^{\frac{1}{h}}};$$

- This q, is independent of the length of time (or cycles, N_w) considered

INTERGRAPH

FATIGUE DATA

Use ASME VIII-2 S-N welded fatigue data

- ASME Section VIII Division 2 Annex 3-F, paragraph 3-F.2 provides the number of allowed cycles for welded joints in equation (3-F.4).
- But equation (3-F.4) references an equivalent structural stress range rather than the B31.3 stress range defined in paragraph 319.
- Equation (W-1) includes additional adjustments provided in paragraph 5.5.5 of VIII-2 to produce the allowed number of cycles for a welded joint using the B31.3 expansion stress range formula:

$$N_{ti} = \frac{f_I}{f_E}\left(\frac{CF \cdot f_{M,k} \cdot f_t}{S_{Ei} \cdot T_E{}^k}\right)^m \qquad \text{(W-1)}$$

3-F.2.2

The number of allowable design cycles for the welded joint fatigue curve shall be computed as follows.

(a) The design number of allowable design cycles, N, can be computed from Equation (3-F.4) based on the equivalent structural stress range parameter, $\Delta S_{ess,k}$, determined in accordance with paragraph 5.5.5 of this Division. The constants C and h for use in Equation (3-F.4) are provided in Table 3-F.10. The lower 99% Prediction Interval (-3σ) shall be used for design unless otherwise agreed to by the Owner-User and the Manufacturer.

$$N = \frac{f_I}{f_E}\left(\frac{f_{MT} \cdot C}{\Delta S_{ess,k}}\right)^{\frac{1}{h}} \qquad \text{(3-F.4)}$$

INTERGRAPH

Reformulation

$$N_{ti} = \frac{f_I}{f_E}\left(\frac{CF \cdot f_{M,k} \cdot f_t}{S_{Ei} \cdot T_E{}^k}\right)^m \qquad \text{(W-1)}$$

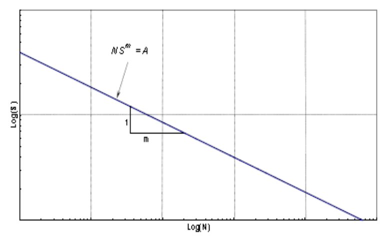
- Re-writing (W-1) in the form $N = aS^{-m}$, you can show:

$$a = \frac{f_I}{f_E} \cdot \left(\frac{CF \cdot f_{M,k} \cdot f_t}{T_E{}^k}\right)^m \qquad \text{(W-9)}$$

- So:

$$N_{ti} = a \cdot S_{Ei}{}^{-m}$$

Characteristic S-N curve



Total wave damage

- Total wave damage is:

$$D = \frac{N_d}{A} q^m \Gamma\left(\frac{m}{h} + 1\right)$$

- Evaluating q with $S_R = S_{Ei}$ (the B31.3 expansion stress range associated with the 100 year storm wave height), and N_d as the design number of cycles, the accumulated wave fatigue damage, D, is:

$$D = \frac{N_d}{a}\left[\frac{S_{Ei}}{(\ln(N_w))^{\frac{1}{h}}}\right]^m \Gamma\left(\frac{m}{h} + 1\right)$$

Setting the allowed stress for the maximum wave damage

- This fatigue damage due to wave loads d_w plus the fatigue damage from other sources d_t cannot exceed 1.0. Therefore, d_w is remaining life after other, non-wave displacement cycles (d_t)

$$D = d_w = 1 - d_t \qquad D = \frac{N_d}{a}\left[\frac{S_{Ei}}{(\ln(N_w))^{\frac{1}{h}}}\right]^m \Gamma(\frac{m}{h}+1)$$

- Solve for S_{Ei} and set that as your "allowable maximum probable stress range during N_d wave cycles". N_d is the number of design life cycles for the system (e.g., cycles over a 20 year life).

$$S_{aw} = \left(\frac{d_w \cdot a}{N_d}\right)^{\frac{1}{m}} \cdot \frac{(\ln(N_w))^{\frac{1}{h}}}{\Gamma(\frac{m}{h}+1)^{\frac{1}{m}}} \qquad \text{(W-8)}$$

- where:

$$a = \frac{f_I}{f_E} \cdot \left(\frac{CF \cdot f_{M,k} \cdot f_t}{{T_E}^k}\right)^m \qquad \text{(W-9)}$$

INTERGRAPH

All terms are defined

- So, as long as the stress range associated with the maximum expected wave height (e.g., the100 year storm height) is below S_{aw}, fatigue failure is not predicted.

$$S_{aw} = \frac{1}{C_{ME}} \cdot \left(\frac{d_w \cdot a}{N_d}\right)^{\frac{1}{m}} \cdot \frac{(\ln(N_w))^{\frac{1}{h}}}{\Gamma(\frac{m}{h}+1)^{\frac{1}{m}}} \qquad \text{(W-8)}$$

  - where:

$$a = \frac{f_I}{f_E} \cdot \left(\frac{CF \cdot f_{M,k} \cdot f_t}{{T_E}^k}\right)^m \qquad \text{(W-9)}$$

$$d_w = 1 - d_t \qquad \text{(W-5)}$$

- In this manner, the stress range need only be calculated for the 100 year storm wave height and the accumulate wave damage will be estimated using the Weibull stress range distribution.

INTERGRAPH

AN EXAMPLE

Example

- Given
  - Units: Metric
  - Material: Ferritic Steel
  - T-bar: 9.525 mm
  - Stress range is less than yield
  - Pipe will be in seawater and will have no cathodic protection

INTERGRAPH

Example

$$a = \frac{f_I}{f_E} \cdot \left(\frac{CF \cdot f_{M,k} \cdot f_t}{T_E{}^k} \right)^m \qquad \text{(W-9)}$$

- Fatigue Improvement Factor (ASME VIII-2) f_I=1.0
- Environmental Correction Factor (Table W302.2) f_E=3.0 (seawater with free corrosion)
- Welded Joint Fatigue Curve Coefficient (Table W302.1a) CF=14137
- Fatigue Factor for stress ratio $f_{M,k}$=1.0
- Temperature correction factor f_t =1.0
- Effective component thickness (text in W302.1) TE=16.
- Welded Joint Fatigue Curve Exponent (Table W302.1a) m=3.13
- Fatigue strength thickness exponent (Table W302.1a) k=0.222

$$a = \frac{1}{3} \cdot \left(\frac{14137 \cdot 1 \cdot 1}{16^{0.222}} \right)^{3.13}$$

$$a = 0.475 * 10^{12}$$

INTERGRAPH

Example

$$d_w = 1 - d_t$$

- For this example, let the fatigue damage due to thermal stress with constant amplitude d_t = 0.60

$$d_w = 1 - 0.60$$

$$d_w = 0.40$$

INTERGRAPH

Example

- Design Storm Wave height associated cycles (N_W)

$$N_w = 3.156 \cdot 10^7 \cdot V_o \cdot L_w \quad \text{(W-6)}$$

  - Average zero-crossing frequency in Hertz (typical, see W302.2.1) V_o=0.159 (period of about 6 sec)
  - Design Storm Period of Occurrence in years, L_w=100

$$N_w = 3.156 \cdot 10^7 \cdot 0.159 \cdot 100$$
$$N_w = 501.18 \cdot 10^6$$

- Design number of pipe stress cycles (N_d)

$$N_d = 3.156 \cdot 10^7 \cdot V_o \cdot L_d \quad \text{(W-7)}$$

  - Average zero-crossing frequency in Hertz, V_o=0.159
  - Piping Cyclic Design Life in years, L_d=20

$$N_d = 3.156 \cdot 10^7 \cdot 0.159 \cdot 20$$
$$N_d = 100.04 \cdot 10^6$$

INTERGRAPH

Example

$$S_{aw} = \left(\frac{d_w \cdot a}{N_d}\right)^{\frac{1}{m}} \cdot \frac{(\ln(N_W))^{\frac{1}{h}}}{\Gamma(\frac{m}{h}+1)^{\frac{1}{m}}} \quad \text{(W-8)}$$

  - Allowable Fatigue damage for variable Wave Loadings (above) d_w=0.40
  - Adjusted S-N constant (above) *a*=0.475*1012
  - Design number of pipe stress cycles (above) N_d=100.04*10^6
  - Welded Joint Fatigue Curve Exponent (Table W302.1a) *m*=3.13
  - Design Storm Wave height associated cycles (above) N_W=501.18*10^6
  - Weibull stress range shape distribution parameter (typical, see W302.2.1) *h*=1.0
  - Gamma Function evaluation (Table W301 where [*(m/h)+1*]=4.14) *Γ*(4.14)=7.17
    - $\frac{m}{h} + 1 = \frac{3.13}{1} + 1 = 4.14$

$$S_{aw} = \left(\frac{0.40 \cdot 0.475 \cdot 10^{12}}{100.04 \cdot 10^6}\right)^{\frac{1}{3.13}} \cdot \frac{(\ln(501.18 \cdot 10^6))^{\frac{1}{1}}}{(7.17)^{\frac{1}{3.13}}} = 119 \text{ MPa}$$

INTERGRAPH

Example

- The computed maximum stress range due to wave motion – S_{EW} – shall remain below the allowable maximum probable stress range – S_{aw} – through the expected life of the system.
- Here:
  - S_{EW} is calculated in accordance with B31.3 paragraph 319 for the maximum wave height
  - S_{aw} is 119 MPa
- In this example, the calculated B31.3 expansion stress range caused by maximum probable wave height (trough to peak), S_{EW}, shall not exceed S_{aw} (119 MPa).
- This S_{aw} changes from node to node in the piping system

Other notes of interest in Appendix W

- Applies where the total number of significant cycles exceed 100,000.
- A significant cycle is a stress range that exceeds 20.7 MPa
- Appendix W does not address pressure cycling.
- Integral construction is recommended, fabricated components are not recommended
- An optional (bi-linear) fatigue curve is available for cycle counts above 10 million
- The design Sea State (setting the wave height, wave period and probability density) shall be specified by the owner
- This proposed appendix also has additional requirements regarding fluid service, materials, fabrication, examination and testing

B31.3 302.3.5(D) "WHEN THE COMPUTED STRESS RANGE VARIES" – APPLYING EXISTING B31.3 RULES IN CAESAR II

Questions / Comments?

B31.3 302.3.5(D) "WHEN THE COMPUTED STRESS RANGE VARIES" – APPLYING EXISTING B31.3 RULES IN CAESAR II

Thank you