CAU2015

F=KX

How CAESAR II formulates the global stiffness matrix

INTERGRAPH

Agenda

- Math basics
- Comparing matrix math to CAESAR II
  - Single pipe element
  - Single bend
  - A two pipe system
  - A pipe-bend-pipe system
  - A tee
- Other considerations

Content developed as part of the CAESAR II on-line video training series.

INTERGRAPH

MATH BASICS

To the guided cantilever method

Concept

- What is the thermal load on single pipe between two anchors?
  - 4 inch schedule STD
  - Temperature = 350°C
  - Length undefined (L)

Approach

- What is the thermal load on single pipe between two anchors?
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  - Temperature = 350°C
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- Determine free thermal growth: $\Delta = \alpha L$

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Math Basics

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  - Temperature = 350°C
  - Length undefined (L)
- Determine free thermal growth: $\Delta = \alpha L$
- Calculate load to return the end: $P = K\,\Delta$; $K = AE/L$
- Both anchors see $P = AE\alpha$

- And P is large! (P exceeds 1800kN.)

INTERGRAPH

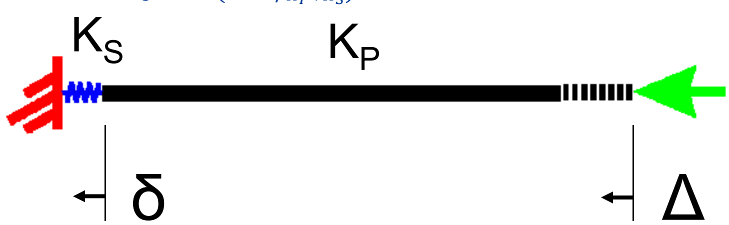
Math Basics

- Add spring at one end to drop the load

INTERGRAPH

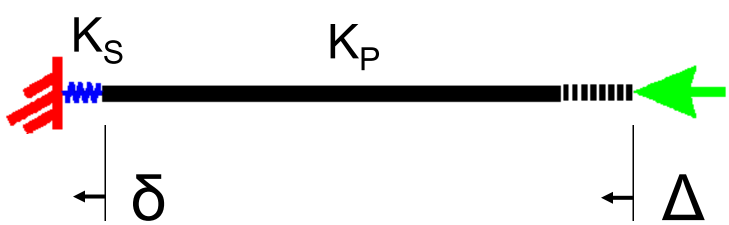
Math Basics

- Add spring at one end to drop the load
- Again, what is the load (P) required to return free growth?
- Monitor two positions: δ, Δ
  - $P = K_P \cdot (\Delta - \delta) = K_s \cdot \delta$
  - $\delta = \left({}^{K_P}/_{K_P + K_S}\right) \cdot \Delta$
  - $P = K_s \cdot \delta = \left({}^{K_P \cdot K_S}/_{K_P + K_S}\right) \cdot \Delta$



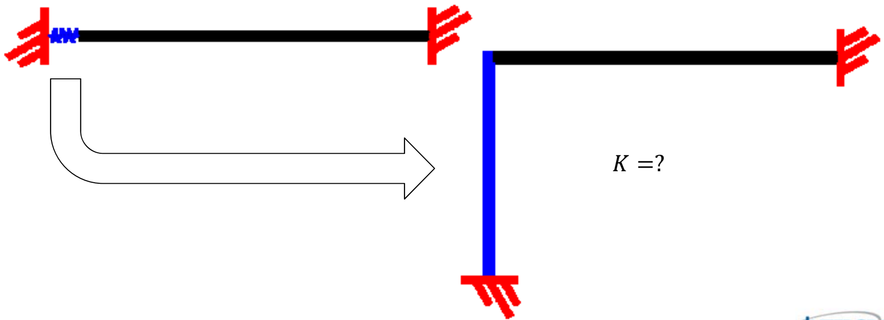
Math Basics

- $P = K_s \cdot \delta = \left({}^{K_P \cdot K_S}/_{K_P + K_S}\right) \cdot \Delta$
  - If $K_S \gg K_P$; ${}^{K_S}/_{K_P + K_S} \rightarrow 1$ and $P = K_P \cdot \Delta$ (our original result)
  - If $K_P \gg K_S$; ${}^{K_P}/_{K_P + K_S} \rightarrow 1$ and $P = K_S \cdot \Delta$ (based solely on K_S!)
- You can dial in any load you wish by adjusting support stiffness!



Math Basics

- Yes, you can dial in any load you wish by adjusting support stiffness
- But seldom is that an option.
- What if that spring was replaced by a cantilever?

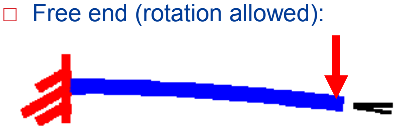


$K = ?$

INTERGRAPH

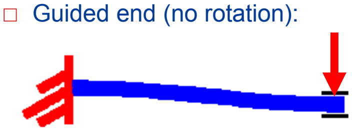
Math Basics

- Cantilever stiffness is a function of the end conditions:
  - Free end (rotation allowed):



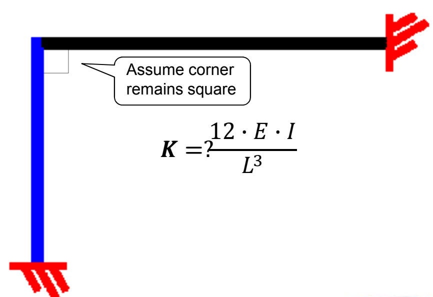
$$K = \frac{3 \cdot E \cdot I}{L^3}$$

  - Guided end (no rotation):



$$K = \frac{12 \cdot E \cdot I}{L^3}$$

(Conservative)

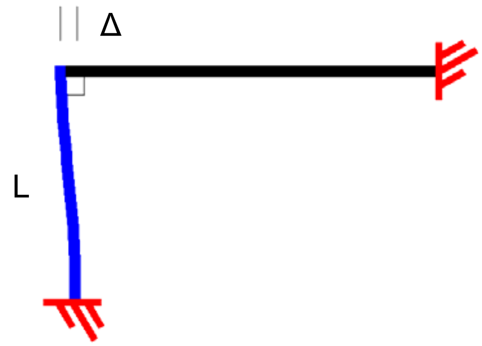


$$\boldsymbol{K} = ?\frac{12 \cdot E \cdot I}{L^3}$$

INTERGRAPH

Math Basics

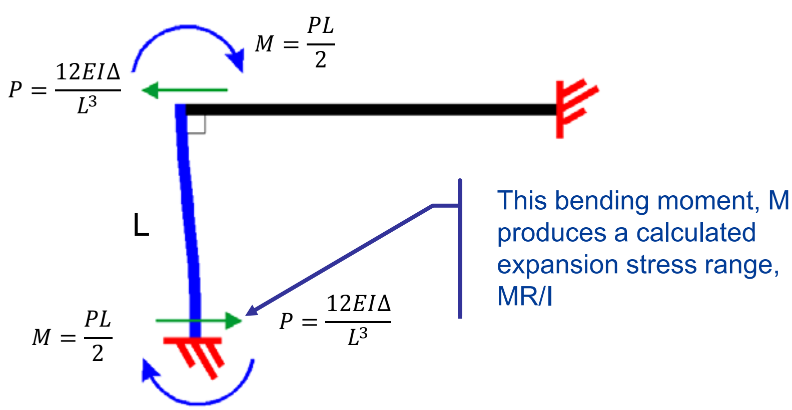
- But things get a little more complicated…



INTERGRAPH

Math Basics

- But things get a little more complicated…



INTERGRAPH

Math Basics

- This leads to the guided cantilever approach
  - With load known, bending can be calculated ($M = \frac{PL}{2} = \frac{6EI\Delta}{L^2}$)
  - With bending known, stress can be calculated ($\sigma = \frac{MR}{I} = \frac{6ER\Delta}{L^2}$)
- For many years, engineers could estimate the magnitude of stress with this simple equation
- Compare this formula with the B31.3 “no formal analysis” equation:

Our stress equation here:

$$\sigma = \frac{6ER\Delta}{L^2}$$

Eqn. (16) para. 319.4.1(c):

$$\frac{Dy}{(L-U)^2} \leq K_1 \quad ; \; K_1 = \frac{constant \cdot S_A}{E_a}$$

$$\frac{DyE_a}{constant \cdot (L-U)^2} \leq S_A$$

INTERGRAPH

Introduction

- Similar to these quick hand calculations, a computer program takes user input describing system layout, restraints and applied load and solves for the position of each point identified in the system
- With final position known, the loads on the distorted element can be calculated
- Pipe stress, then, can be calculated based on these internal forces and moments
- This session will develop the major steps in creating and evaluating this stiffness method for piping system analysis

INTERGRAPH

Flexibility Method – life before the PC

- Beyond the guided cantilever method, larger systems were analyzed using the flexibility method
  - The US Navy's Mare Island program (MEC 21) was one of the first
  - CAESAR II still references the bend formula found in MEC 21
- Flexibility Method – X=AF: position equals flexibility times load
  - Direct solution – know A & F, find X
  - Piping codes continue to offer flexibility factor for elbows
  - Limited to static analysis
    - Dynamic solution works with stiffness (e.g. $\sqrt{K/M}$), not flexibility
    - Auton Computing's conversion from Autoflex to Dynaflex in 1970's
- Stiffness Method – F=KX: load equals stiffness times position
  - Know F & K, find X
  - n equations with n unknowns

INTERGRAPH

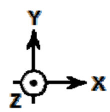
COMPARING MATRIX MATH TO CAESAR II

A Series of Examples

Session Format

- Concepts in F=KX will first be developed.
- These concepts will be expressed in Mathcad
- Followed with a CAESAR II analysis.
- Results will be compared.

- To keep it simple, all models are planar (2-D)
  - Rather than dealing with 6 degrees of freedom for every node, we will be using X, Y & RZ



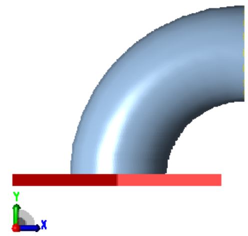
#1 The simple beam element (in 2-D)

- Build and load the stiffness matrix for a single straight pipe
  - Set stiffness terms in Mathcad
  - Build a 2D (planar) beam stiffness matrix for a 4"Std pipe
  - Add anchor at near end
  - Compare with CAESAR II
    - Displace far end
    - Apply loads at far end

CAESAR II Models: 1 ELEMENT PLANAR
1 ELEMENT PLANAR - FORCES

#2 The bend element

- Build and load a 2D bend element
  - What is a flexibility factor

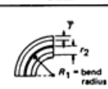


What of this Flexibility Factor?

- Except for elbows, bends and miters, the component flexibility factor is 1.
- For an elbow this factor is 1.65/h:

APPENDIX D
FLEXIBILITY AND STRESS INTENSIFICATION FACTORS

TABLE D300[1]
FLEXIBILITY FACTOR, k AND STRESS INTENSIFICATION FACTOR, i

Description	Flexibility Factor, k	Stress Intensification Factor [Notes (2), (3)] Out-of-Plane, i_o	In-Plane, i_i	Flexibility Characteristic, h	Sketch
Welding elbow or pipe bend [Notes (2), (4)-(7)]	$\frac{1.65}{h}$	$\frac{0.75}{h^{2/3}}$	$\frac{0.9}{h^{2/3}}$	$\frac{\bar{T} R_1}{r_2^2}$	

Flexibility Factor

- Here's a definition from Companion Guide to the ASME Boiler & Pressure Vessel Code Volume One:

  "The flexibility factor is the length of straight pipe having the same flexibility as the component divided by the centerline length of the component."

- Flexibility, here, refers to the angle of rotation for a given bending moment (with no control of lateral deflection)

INTERGRAPH

Flexibility Factor for an Elbow

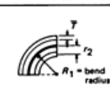
- B31.3 Appendix D provides an elbow flexibility characteristic as:

$$h = \overline{T} \cdot R_1 / {r_2}^2$$

- The flexibility factor for this elbow is:

$$k = 1.65 / h$$

TABLE D300[1]
FLEXIBILITY FACTOR, *k* AND STRESS INTENSIFICATION FACTOR, *i*

Description	Flexibility Factor, k	Stress Intensification Factor [Notes (2), (3)] Out-of-Plane, i_o	Stress Intensification Factor [Notes (2), (3)] In-Plane, i_i	Flexibility Characteristic, h	Sketch
Welding elbow or pipe bend [Notes (2), (4)-(7)]	$\frac{1.65}{h}$	$\frac{0.75}{h^{2/3}}$	$\frac{0.9}{h^{2/3}}$	$\frac{\overline{T} R_1}{{r_2}^2}$	

INTERGRAPH

Flexibility Factor Example

- A 4 inch, standard wall, long radius elbow will have the following terms:

$$\overline{T} = 6.0198mm \ , R_1 = 152.4mm, r_2 = 54.14mm$$

- Therefore flexibility factor for this elbow is:

$$k = 5.272$$

- The arc length of this elbow is:

$$\mathrm{L} = {}^{\pi}/_{2} \cdot R_1 = 239.4mm$$

- So a straight pipe that is $k \cdot L = 1262mm$ long should have the same end rotation as this elbow.

INTERGRAPH

Flexibility Factor Example

- A 4"STD straight pipe that is 1262mm long should have the same end rotation as its long radius elbow:

Node	DX mm.	DY mm.	RZ deg.
10	-0.000	-0.000	0.0000
19	-0.053	0.031	0.0600
20	-0.116	0.203	0.1201
30	-0.116	0.203	0.1201
100	0.000	0.000	0.0000
110	0.000	1.323	0.1201

Elbow

Straight

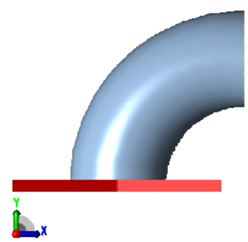
Q.E.D.

CAESAR II Model: FLEX CHECK

INTERGRAPH

The bend element

- Build and load a 2D bend element
  - Build a (local) flexibility matrix
  - Check flexibility matrix in CAESAR II
  - Generate a stiffness matrix from the flexibility matrix
  - Add anchor
  - Compare with CAESAR II
    - Displace far end (F=KX)
    - Apply loads on far end (X=AF)

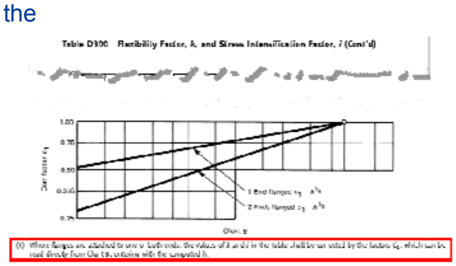


CAESAR II Model: BEND

INTERGRAPH

The bend element

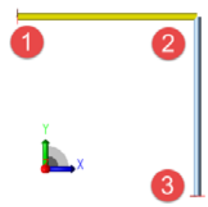
- Build and load a 2D bend element
  - Build a (local) flexibility matrix
  - Check flexibility matrix in CAESAR II
  - Generate a stiffness matrix from the flexibility matrix
  - Add anchor
  - Compare with CAESAR II
    - Displace far end (F=KX)
    - Apply loads on far end (X=AF)
  - Other discussion
    - B31.3 Appx. D provides an adjustment for stiffeners (flanges) that restrict ovalization
    - CAESAR II provides a scratchpad to review these values



INTERGRAPH

#3 Assembling elements

- Build a two element system; apply thermal strain
  - Rotate a second beam (local to global)
    - The transformation matrix
  - Assemble global element stiffness matrix
  - Add anchors at both ends
  - Calculate “self-load” due to thermal strain
  - Assemble global load vector
  - Compare with CAESAR II
    - Use X=AF to solve for corner position
    - Solve for internal forces and moments

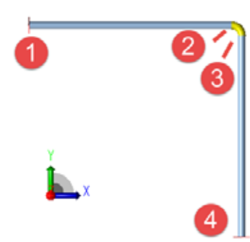


CAESAR II Model: 2 ELEMENT PLANAR

INTERGRAPH

More elements in series

- Build a straight-bend-straight “system”
  - Build element stiffness matrices for beams and bend
  - Rotate bend and second beam
  - Assemble global stiffness matrix from elements
  - Add anchors as boundary conditions
  - Calculate/set thermal load for each element
  - Assemble global load vector
  - Compare with CAESAR II
    - Use X=AF to find position of internal nodes
    - Calculate element forces and moments using thermal position

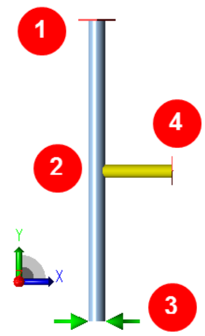


CAESAR II Model: STRAIGHT-BEND-STRAIGHT

INTERGRAPH

#4 Branches

- Build a branched "system"
  - Build local stiffness matrices for beams
  - Rotate run pipe to global system
  - Assemble stiffness matrix from elements – focus on offset applied to branch
  - Add boundary conditions (anchors and restraint)
  - Calculate/set thermal load for each element
  - Build load vector – offset vector position
  - Compare with CAESAR II
    - Use X=AF to find position of internal nodes
    - Calculate element forces and moments using thermal position
  - Other points
    - Bandwidth



CAESAR II Model: TEE

INTERGRAPH

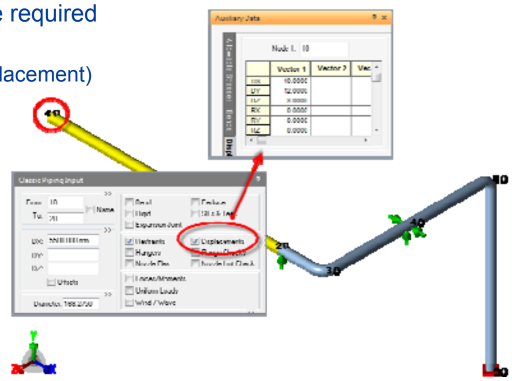
OTHER CONSIDERATIONS

Additional boundary conditions

- Displacements
  - Add rigid stiffness for each displacement direction
  - Update the load vector to include enough force to produce the required displacement/rotation
    - F=(restraint stiffness)·(displacement)
    - Here:

$$K_{rigid\_trans} = 1.75 \cdot 10^{11}\, {}^{N}/_{mm}$$
$$K_{rigid\_rot} = 1.13 \cdot 10^{11}\, {}^{N \cdot m}/_{deg}$$
$$F_{10x} = K_{trans} \cdot d_{10x}$$
$$F_{10y} = K_{trans} \cdot d_{10y}$$
$$F_{10z} = K_{trans} \cdot d_{10z}$$

(with enough load, you can move an anchor anywhere)



INTERGRAPH

Additional boundary conditions

- Displacements
- Nonlinear restraints
  - Initially, include stiffness of all nonlinear restraints (double-acting)
  - Calculate position of pipe and loads on these restraints
  - If load on the nonlinear restraint is in the correct direction, OK – for that restraint – it is “active” for the load case
    - For example, a -Y load is calculated on a +Y restraint
  - If load on the nonlinear restraint is improper – it is “inactive” for this case
    - Remove that restraint stiffness and now monitor position
  - If any nonlinear stiffness assumption proves wrong, reanalyze with the update stiffness matrix and updated load vector (if required)
  - Continue testing all nonlinear conditions until results are consistent for the input:
    - Monitor load on active restraints
    - Monitor displacement on inactive restraints

INTERGRAPH

Additional boundary conditions

- Displacements
- Nonlinear restraints
- Friction (stick/slip)
  - Initial condition (assume stick) – Add translational restraints perpendicular to the restraint vector with coefficient of friction (μ) specified
  - If restraint load on that pair of added restraints is less than the resisting force, μN, model is valid
  - If restraint load on that pair of added restraints is greater than the resisting force, μN, model is updated – remove restraints and provide a friction force, μN, opposite the vector loading those remove restraints

INTERGRAPH

Additional boundary conditions

- Displacements
- Nonlinear restraints
- Friction (stick/slip)
- CNodes
  - Consider as a "partial" element with connection based on restraint type
  - For example:
    - Matrix at right shows the planar (X,Y,RZ) stiffness matrix for a 4 node system
    - If Node 1 was rigidly connected to Node 4 in the X direction, then
    - Add rigid stiffness in X between 1 & 4
    - (Other cells in 1:4 and 4:1 remain empty)

1:1	1:2		1:4
2:1	2:2	2:3	
	3:2	3:3	3:4
4:1		4:3	4:4

INTERGRAPH

A note on load vectors

- Primitive loads are used to build CAESAR II load cases
- Each primitive provides a load vector component
- Load cases combine these primitive loads
- For example: Load Case 1: W+P1+T1+D1 (OPE)
  - Individual load vectors are assembled (here, w1 is the x,y,z,rx,ry,rz weight load assigned to node 1)

$$W=\begin{vmatrix} w1 \\ w2 \\ w3 \\ - \\ wn \end{vmatrix} \quad P=\begin{vmatrix} p1 \\ p2 \\ p3 \\ \cdots \\ pn \end{vmatrix} \quad T=\begin{vmatrix} t1 \\ t2 \\ t3 \\ \cdots \\ tn \end{vmatrix} \quad D=\begin{vmatrix} d1 \\ d2 \\ d3 \\ \cdots \\ dn \end{vmatrix}$$

  - The load vector used with the global stiffness matrix, then, is:

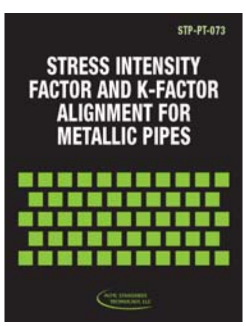
$$F=\begin{vmatrix} w1 \\ w2 \\ w3 \\ \cdots \\ wn \end{vmatrix}+\begin{vmatrix} p1 \\ p2 \\ p3 \\ \cdots \\ pn \end{vmatrix}+\begin{vmatrix} t1 \\ t2 \\ t3 \\ \cdots \\ tn \end{vmatrix}+\begin{vmatrix} d1 \\ d2 \\ d3 \\ \cdots \\ dn \end{vmatrix} \quad \text{or} \quad F=\begin{vmatrix} w1+p1+t1+d1 \\ w2+p2+t2+d2 \\ w3+p3+t3+d3 \\ \cdots \\ wn+pn+tn+dn \end{vmatrix}$$

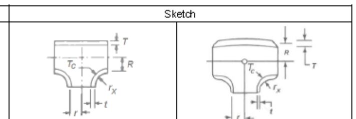
  - Solve for X in F=KX

INTERGRAPH

Flexibility at branch connections

- STP-PT-073: Stress Intensity Factor and k-Factor Alignment for Metallic Pipes
  - Table 1 – Flexibility and Stress Intensification Factors (Sketch No. 2+)
  - Nonmandatory Appendix D – Calculating Flexibility Factors for Branch Connection Models in

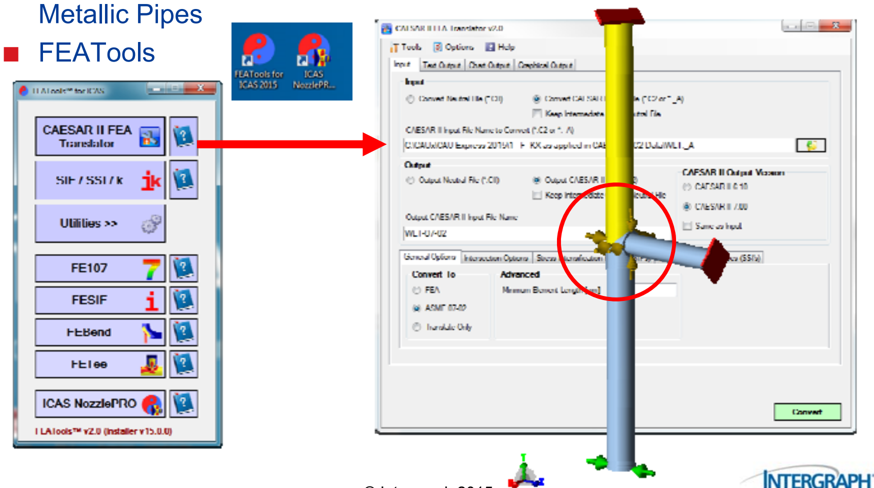


Sketch No. 2.1	Sketch
Welding tee per ASME B16.9 [Notes (3),(9),(10)]	
Term	**Equation**
Run In-plane Flexibility Factor, k_{Ir}	$0.18\,(R/T)^{0.8}\,(d/D)^{5}$
Run Out-of-plane Flexibility Factor, k_{or}	1
Run Torsional Flexibility Factor, k_{tr}	$0.08\,(R/T)^{0.91}\,(d/D)^{5.7}$
Branch In-plane Flexibility Factor, k_{b}	$(1.91(d/D) - 4.32(d/D)^2 + 2.7(d/D)^3)\,(R/T)^{0.77}\,(d/D)^{0.47}\,(t/T)$
Branch Out-of-plane Flexibility Factor,	$(0.34\,(d/D) - 0.49(d/D)^2 + 0.18(d/D)^3)\,(R/T)^{1.46}\,(t/T)$
Branch Torsional Flexibility Factor, k_{tb}	$(1.08(d/D) - 2.44(d/D)^2 + 1.52(d/D)^3)\,(R/T)^{0.77}\,(d/D)^{1.61}\,(t/T)$
Run SIF In-plane, i_{Ir}	$0.98\,(R/T)^{0.35}\,(d/D)^{0.72}\,(t/T)^{-0.52}$
Run SIF Out-of-plane, i_{or}	$0.61\,(R/T)^{0.29}\,(d/D)^{1.96}\,(t/T)^{-0.53}$
Run SIF Torsional, i_{tr}	$0.34\,(R/T)^{2/3}\,(d/D)\,(t/T)^{-0.5}$
Branch SIF In-plane, i_{Ib}	$0.33\,(R/T)^{2/3}\,(d/D)^{0.18}\,(t/T)^{0.7}$
Branch SIF Out-of-plane, i_{ob}	$0.42\,(R/T)^{2/3}\,(d/D)^{0.37}\,(t/T)^{0.37}$
Branch SIF Torsional, i_{tb}	$0.42\,(R/T)^{2/3}\,(d/D)^{1.1}\,(t/T)^{1.1}$

INTERGRAPH

Flexibility at branch connections

- STP-PT-073: Stress Intensity Factor and k-Factor Alignment for Metallic Pipes
- FEATools



INTERGRAPH

F=KX AS APPLIED IN CAESAR II

Questions / Comments

More Information…

- Again, this content was initially developed as part of the CAESAR II on-line video training series. (Visit www.pipingdesignonline.com)
- Along with the (captioned) videos, there is a workbook for the many exercises.

INTERGRAPH

F=KX AS APPLIED IN CAESAR II

Thank you